30625

VI Semester B.Sc. Degree Examination, September/October 2020

MATHEMATICS

Paper XII (6.1) – Trigonometry, Topology and Fuzzy Sets

Time : 3 Hours Max. Marks : 80

Instructions : *Answer* ***all*** *Sections.*

SECTION – A

Answer **any ten** of the following : **(10 × 2 = 20)**

1. Define a Topology on a set with example.
2. Show that an open interval is an open set in (R, u).
3. Prove that A singleton set $\{x\}$ is not an open set in $\{R, u\}$.
4. Prove that in a discrete topology (X, τ) every subset of X is closed.
5. Give an example to show that any union of closed sets need not be closed.
6. If A is closed and B is open then prove that $A - B$ is closed.
7. Define derived set with example.
8. Prove that $\sinh(ix) = i \sin x$.
9. Prove that $\sinh 3x = 3\sinh x + 4\sinh^3 x$.
10. Separate Real and Imaginary parts of $\sin(x + iy)$.
11. Define union and intersection of two fuzzy subsets.
12. Define α-cut and strong α-cut set of fuzzy subset A with an example each.

SECTION – B

Answer **any five** of the following : **(5 × 6 = 30)**

13. Let X be any set, τ be family of subsets of X defined as follows.

    A subset G of X belongs to τ ie $G \in \tau$ iff, (a) G is empty 'OR' (b) G' is finite then, prove that τ is a topology on X.

 P.T.O.

14. Let (X,τ) be a topological space, Let $A, B \subset X$ then prove that, (a) $A \subset \overline{A}$ (b) $\overline{A}$ is closed (c) $\overline{A}$ is the smallest closed set containing A (d) A is closed iff $A = \overline{A}$.

15. Let (X, τ) be a topological space A and B are subsets of X then prove that (a) $d(\phi) = \phi$ (b) If $A \subset B$ then $d(A) \subset d(B)$.

16. Let (X, τ) be a topological space, A and B are subsets of X then (a) $A^\circ \subset A$ (b) A° is the union of all open sets contained in A and hence it is an open set (c) A° is the largest open set contained in A if $B \subset A$ and B is open then $B \subset A^\circ$ (d) A is open iff $A = A^\circ$.

17. Let (X,τ) be a topological space and $A \subset X$, which is neither empty nor singleton, Find $d(A)$.

18. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, X, \{a\}, \{a,b\}, \{a, b, c\} \{a, b, c, d\}\}$ be a topology of X. Find A°, $(A')^\circ$, $\partial(A)$ where $A = \{a, c, d\}$.

19. Prove that every finite T_1-space is discrete space.

SECTION – C

Answer **any five** of the following : **(5 × 6 = 30)**

20. Show that $\operatorname{cosec}(ix) = -i \operatorname{cosec} h\, x$.

21. Expand $\cos 8\theta$ interms of powers of $\sin\theta$ and $\cos\theta$.

22. Find Real and Imaginary parts of $\sin(x+iy)$.

23. Sum the series

$$1 + \frac{\cos\theta}{1!} + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots \text{ upto } \infty.$$

24. Let us prove that

$$\log\left[\frac{\cos(x-iy)}{\cos(x+iy)}\right] = zi \tan^{-1}[\tan x \tanh y].$$

25. Let A and B two fuzzy subsets of X. Let $\alpha, \beta \in [0, 1]$ then prove that

    (a) $\alpha_{(A \wedge B)} = \alpha_A \cap \alpha_B$

    (b) $\alpha_{(A \vee B)} = \alpha_A \cup \alpha_B$

26. Let $X = \{a, b, c, d, e\}$, $A = \{(a, 0), (b, 0, 2), (c, 0, 6), (d, 1), (e, 0, 5)\}$.

    Find all α-cutsets and strong α-cutsets of A where $\alpha = 0.4, 0.2, 1$.
