30621

VI Semester B.Sc. Degree Examination, September/October 2020

MATHEMATICS

Paper XII (6.1) – Trigonometry and Complex Analysis

(Very Old)

Time : 3 Hours Max. Marks : 80

Instructions : *Answer **all** Sections.*

SECTION – A

Answer **any ten** of the following : **(10 × 2 = 20)**

1. Show that : $\operatorname{cosec}(ix) = -i \operatorname{cosech} x$.

2. Prove that : $\sinh 3x = 3\sinh x + 4\sinh^3 x$.

3. Prove that : $\cos(x-y) = \cosh x \cosh y - \sinh x \sinh y$.

4. Find the general value of $\sqrt{3} + i$.

5. Find whether $f(z) = \sin z$ is differentiable at $z_0 = i$.

6. Find the fixed points of the transformation $W = 3z - 4/z$.

7. Show that $u = x^2 - y^2 + x + 1$ is harmonic.

8. Evaluate : $\int_C \frac{1}{z(z-1)}\, dz$ where C is $|z| = 3$.

9. Evaluate : $\int_C (\bar{z})^2\, dz$ around the circle $|z-1| = 1$.

10. Show that $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$ represents a Line through the origin.

11. Evaluate : $\lim_{z \to i}(z^3 - 2z^2 + 5z)$

12. Evaluate : $\int_0^{3+i} z^2\, dz$ along the line $3y = x$.

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SECTION – B

Answer **any five** of the following : **(5 × 6 = 30)**

13. Show that $\frac{\cos 7\theta}{\cos\theta} = 64\cos^6\theta - 112\cos^4\theta + 56\cos^2\theta - 7$.

14. Find the real and imaginary parts of $\tan x + iy$.

15. Prove the necessary condition for a function $f(z)$ to be analytic and establish them.

16. Prove that $f(z) = \cosh z$ is analytic and $f'(z) = \sinh z$.

17. State and prove Cauchy's integral formula.

18. Show that the transformation $w = 1/z$ transform a circle to a circle 'OR' to a straight line.

19. Construct the Analytic function $f(z) = u + iv$ given $u + v = e^x(\sin y + \cos y)$.

SECTION – C

Answer **any five** of the following : **(5 × 6 = 30)**

20. If $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$ then find the corresponding analytic function $f(z)$.

21. If $u - v = (x - y)(x^2 + 4xy + y^2)$ find $f(z) = u + iv$ is an analytic function of z, Find $f(z)$ interms of z.

22. If a function $f(z) = u + iv$ is analytic in a domain D and $|f(z)|$ is constant, show that $f(z)$ is also a constant in D.

23. Evaluate $\int_0^{a+i}(x^2 - iy)\,dz$ along the curve $y = x$ and $y = x^2$.

24. Evaluate $\int_C \bar{z}\, dz$ where C is given by two lines joining $z = 0$, $z = 2i$ then $z = 2i$ to $z = 4 + 2i$.

25. If $f(z) = u + iv$ is analytic and ϕ is any differentiable function of x and y, show that

$$\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 = \left[\left(\frac{\partial \phi}{\partial u}\right)^2 + \left(\frac{\partial \phi}{\partial v}\right)^2\right]\left|\frac{1}{f(z)}\right|^2$$

26. Evaluate $\int_C (x^2 - iy^2)dz$ along $y = 2x^2$ from (1, 2) to (2, 8).