15613 – A

Sixth Semester B.Sc. Degree Examination, May 2017
MATHEMATICS – XIV(A)
Theory of Graphs (Old)

Time : 3 Hours Max. Marks : 80

Instruction : *Answer **all** Sections.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Define bridge and block of a graph.
2. Draw two forests with four vertices.
3. Define non separable graph with example.
4. Show that if Cp is a cycle with $p \geq 3$, then $\lambda\, C(p) = 2$.
5. Show that the complete bipartite graph $K_{3,3}$ is Hamiltonian graph but not Eulerian graph.
6. If a tree has 2020 vertices, then find the sum of the degrees of the vertices.
7. Construct a graph G satisfying

   $K(G) = 1,\ \lambda\ (G) = 3$ and $\delta(G) = 4$.
8. Define binary tree. Give an example of a binary tree with five vertices.
9. State Menger's theorem.
10. Show that every Hamiltonian graph is 2-connected.
11. Define Eulerian trail with an example.
12. For which positive integers m and n, $k_{m,n}$ is Eulerian ?

P.T.O.

SECTION – B

Answer **any five** of the following : **(5x6=30)**

13. Prove that a vertex v of a connected graph G is a cut vertex of G if and only if there exist vertices 'u' and 'w' distinct from 'v' such that 'v' is on every u-w path of G.

14. Prove that an edge e of a graph G is a bridge if and only if e is on no cycle of G.

15. Prove that a (p, q) graph G is a tree if and only if G is acyclic and $p = q + 1$.

16. Prove that the number of vertices in a binary tree is odd.

17. If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5, find the number of leaves in T.

18. Let F be a forest with K components (trees). If n is the number of vertices and m is the number of edges of F, then prove that $n = m + k$.

19. For any graph G prove that

    $K(G) \leq \lambda(G) \leq \delta(G)$

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SECTION – C

Answer **any five** of the following : **(5x6=30)**

20. Prove that the following statements are equivalent for a connected graph G.

    i) G is Eulerian.

    ii) Every vertex of G is even degree.

    iii) The set of edges of G can be partitioned in to cycles.

21. If u and v are distinct non-adjacent vertices of a graph G with p vertices such that $\deg u + \deg v \geq p$ then prove that the graph G + uv is Hamiltonian if and only if G is Hamiltonian.

22. Show that every simple K-regular graph with $(2k - 1)$ vertices is Hamiltonian.

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23. Draw the graphs.

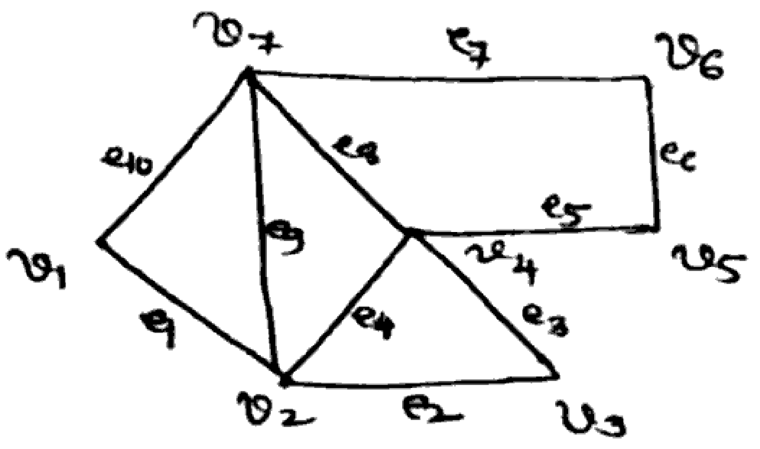
    1) Graph with both Eulerian circuit and Hamiltonian cycle.

    2) Graph with Eulerian but not Hamiltonian.

    3) Graph which is neither Hamiltonian nor Eulerian.

24. Show that a connected graph with exactly two vertices of odd degree has an Eulerian trail.

25. Show that the graph G shown below is Eulerian and find a partition of edges of G into cycles.



G

26. Explain Konigberg's seven bridge problem.