



VI Semester B.Sc. Degree Examination, September/October 2020

MATHEMATICS – XIV

Paper 6.3 – Graph Theory – II

(CBCS – New)

Time : 3 Hours

Max. Marks : 70

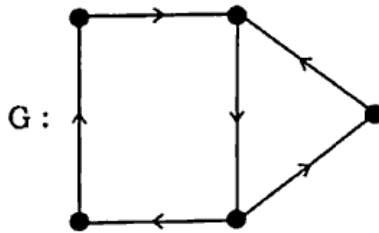
Instructions : Answer all Sections.

SECTION – A

Answer **any five** of the following :

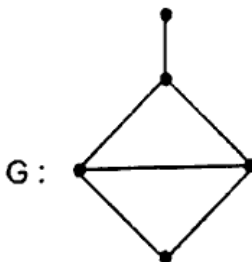
(5 × 2 = 10)

1. Draw line graph of K_3 and define line graph.
2. Draw total graph of P_3 .
3. Define r -regular digraph and draw 1-regular digraph.
4. Find out degree and indegree of the graph G given below :



5. If G is a 4 degree connected planar graph having 16 edges, find the number of regions.
6. Show that $K_{2,4}$ is planar.
7. Define proper coloring of graph and give two different proper coloring of the graph.

G given below



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SECTION - B

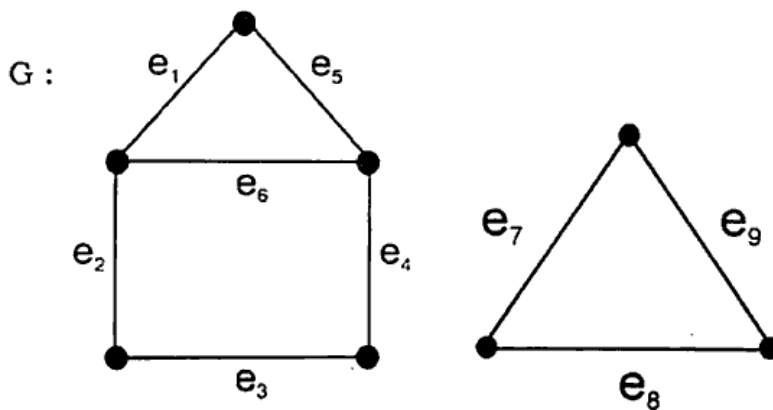
Answer any five of the following :

(5 × 6 = 30)

8. Find the incidence matrix of K_4 .
9. Define incidence matrix and find the graph whose incidence matrix is

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

10. Define cycle matrix and find the cycle matrix of G shown below :



11. If G is a (p, q) graph whose vertices have degree d_i then show that $L(G)$ has q vertices and q_L edges where $q_L = \frac{1}{2} \sum_{i=1}^p d_i^2 - q$.
12. If G is a connected planar graph with p vertices and q edges, if G is triangle free then show that
(a) $q \geq 2r$ (b) $q \leq 2p - 4$.
13. Suppose we wish to supply three houses each with three utilities electricity water and gas. Is it possible to connect each utility with each of three houses without crossing of utility lines?
14. Let G be connected planar graph with p vertices, q edges and r -regions (faces) then prove that $q - p + 2 = r$.



SECTION - C

Answer **any five** of the following :

(5 × 6 = 30)

15. Show that a graph with atleast one edge is bipartite if and only if it is 2-chromatic.
16. Define chromatic number. Find chromatic number of cycle C_p with $p \geq 3$ and hence write the value of $\chi(C_4)$.
17. Prove that every connected simple planar graph G is 6-colorable.

18. Define Digraph. Let D be a digraph of order p and size q with $V(D) = \{V_1, V_2, \dots, V_p\}$ then prove that

$$\sum_{i=1}^p \text{od } V_i = \sum_{i=1}^p \text{id } V_i = q.$$

19. Show that a graph with p vertices is a complete graph if and only if its chromatic polynomial is $f(K_p, \lambda) = \lambda(\lambda-1)(\lambda-2)\dots(\lambda-p+1)$.
20. If $\Delta(G)$ is the maximum of the degrees of the vertices of a graph G , then prove that $\chi(G) \leq 1 + \Delta(G)$.
21. Define chromatic polynomial and with usual notation show that

$$f(G, \lambda) = C_1 \frac{\lambda}{1!} + C_2 \frac{\lambda(\lambda-1)}{2!} + \dots + C_p \frac{\lambda(\lambda-1)\dots(\lambda-p+1)}{p!}$$
