



V Semester B.Sc. Degree Examination, November/December 2019

MATHEMATICS

Paper IX (5.1) – Integral Transforms

(New)

Time : 3 Hours

Max. Marks : 80

Instructions : Answer all the Sections.

SECTION – A

Answer any ten of the following : **(10 × 2 = 20)**

1. Find $L[e^{at}]$.
2. Find $L[\cos^2 4t]$.
3. Find $L[e^{3t} \cos 3t]$.
4. Find $L^{-1}\left[\frac{s}{(s+2)^2}\right]$.
5. Using Convolution theorem find $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$.
6. If $f(x) = x$ in $(-\pi, \pi)$ find Fourier coefficient of a_n .
7. If $f(x) = \begin{cases} -k & \text{in } (-\pi, 0) \\ k & \text{in } (0, \pi) \end{cases}$ find Fourier coefficient of a_0 .
8. Define inverse Fourier cosine and sine transform.
9. Find sine transform of $2e^{-5x} + 5e^{-2x}$.
10. Define Z-transform and inverse Z-transform.
11. If U_n and V_n be any two discrete valued function then $Z_T(C_1U_n + C_2V_n) = C_1Z_T(U_n) + C_2Z_T(V_n)$ where C_1 and C_2 are constant.
12. Find the Inverse Z-transform of $\frac{z}{(z-2)(z-3)}$.

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SECTION - B

Answer **any five** of the following : **(5 × 6 = 30)**

13. Evaluate $L\left[\frac{2\sin 5t \cos 3t}{t}\right]$.
14. Evaluate $L^{-1}\left[\frac{s+2}{s^2 - 4s + 13}\right]$.
15. Verify the Convolution theorem for the function $f(t) = \cos t$, $g(t) = e^t$.
16. Solve $y'' + 9y = 25e^{4t}$ given $y(0) = 3$ and $y'(0) = 1$ using Laplace transform.
17. Solve the simultaneous differential $\frac{dx}{dt} + 4y = 0$; $\frac{dy}{dt} - 9x = 0$, given $x(0) = 2$, $y(0) = 1$ by using Laplace transform.
18. Find the Fourier series of $f(x) = x - x^2$ in $-\pi \leq x \leq \pi$. Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots$.
19. Obtain the Fourier series of $f(x) = |x|$ in $(-L, L)$.

SECTION - C

Answer **any five** of the following : **(5 × 6 = 30)**

20. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$.
21. Find the Fourier sine transform of the function $f(x) = \begin{cases} \sin x & 0 < x < a \\ 0 & x > a \end{cases}$.
22. Find the Fourier cosine transform of $f(x) = e^{-ax} \cos ax$.
23. Using Parseval identity show that $\int_0^\infty \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}$.



24. Given $Z_T(U_n) = \frac{2Z^2 + 3Z + 4}{(Z - 3)^3}$, $|Z| > 3$ show that $U_2 = 21$.

25. Obtain the inverse Z-transform of $\frac{Z}{(Z + 1)^2}$ by Power expansion.

26. Solve by using Z-transforms $y_{n+2} - 4y_n = 0$ given that $y_n = 0$ and $y_1 = 2$.
