



**Fifth Semester B.Sc. Degree Examination, May/June 2016**  
**MATHEMATICS – X**  
**Paper – 5.2 : Differential Equations – II**

Time : 3 Hours

Max. Marks : 80

**Instruction : Answer all the Sections.**

**SECTION – A**

**(10×2=20)**

Answer any ten of the following.

1. Show that  $P_n(1) = 1$ .
2. Define Legendre differential equation and write down the series solution of it.
3. If  $f(x)$  is a polynomial of degree less than 'm', show that  $\int_{-1}^1 f(x) P_m(x) dx = 0$ .
4. Show that  $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ .
5. Show that  $J_2 = J_0'' - \frac{1}{x} J_0'$ .
6. Verify the condition for integrability  $3x^2 dx + 3y^2 dy - (x^3 + y^3 + e^{2z}) dz = 0$ .
7. Solve  $yz dx - 2xz dy + (xy - zy^3) dz = 0$ .
8. Solve  $\frac{dx}{z^2 y} = \frac{dy}{z^2 x} = \frac{dz}{xy^2}$ .
9. Form the partial differential equation from  $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ .
10. Solve  $p^2 + q^2 = 1$ .
11. Solve  $pq = 1$ .
12. Define Clairaut's equation and solve  $z = px + qy + (p^2 + q^2)$ .

P.T.O.



SECTION - B

Answer any five of the following.

(5×6=30)

13. Prove that  $nP_n(x) = x P_n'(x) - P_{n-1}'(x)$ .

14. Prove that  $\int_{-1}^1 P_m(x)P_n(x) dx = \begin{cases} 0 & \text{If } m \neq n; \\ \frac{2}{2n+1} & \text{If } m = n \end{cases}$

15. Show that  $2n J_n(x) = x [J_{n+1}(x) + J_{n-1}(x)]$ .

16. Show that  $\int J_3(x) dx = -J_2(x) - \frac{2}{x} J_1(x)$ .

17. Show that  $x^n J_n(x)$  is a solution of  $xy'' + (1 - 2n)y' + xy = 0$ .

18. Solve,  $(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$ .

19. Solve,  $\frac{dx}{y^2 + yz + z^2} = \frac{dy}{z^2 + zx + x^2} = \frac{dz}{x^2 + xy + y^2}$ .

SECTION - C

Answer any five of the following.

(5×6=30)

20. Form the partial differential equation given that  $f(x + y + z, x^2 + y^2 - z^2) = 0$ .

21. Solve :  $z^2 \left( \frac{p^2}{x^2} + \frac{q^2}{y^2} \right) = 1$ .

22. Solve :  $p(1 - q^2) = q(1 - z)$ .

23. Solve :  $yp = 2yx + \log q$ .

24. Obtain the complete Integral and Singular solution of the equation  $z = px + qy + \log pq$ .

25. Find the complete solution of  $pxy + pq + qy = yz$  by Charpit's method.

26. Find the complete solution of  $(p^2 + q^2)y = qz$ . Charpits method.