

B.Sc. V Semester (CBCS) Degree Examination, May/June - 2019

MATHEMATICS

Paper : IX and (5.2) : Applied Mathematics

(New)

Time : 3 Hours

Maximum Marks : 70

Answer all the sections.

Section - A

Answer any ten of the following.

(10×2=20)

1. Define Divergence of a vector function.
2. Find the normal derivative at the point $(-3, 1, -2)$ for $\phi(x, y, z) = x^3 + yz$.
3. If c is any scalar and $\phi(x, y, z)$ is any scalar point function then prove that $\nabla(C\phi) = C\nabla\phi$.
4. Find the constant 'a' such that $f = y(ax^2 + z)i + x(y^2 - z^2)j + 2xy(z - xy)k$ is solenoidal.
5. Prove that $\nabla \cdot (\nabla \times f) = 0$.
6. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's theorem.
7. Define Geodesics and Isoperimetric problems.
8. Solve the variational problem $\delta \int_0^1 (y^2 + x^2 y') dx = 0, y(0) = 0, y(1) = 1$.
9. Define a homogeneous linear partial differential equation of second order.
10. Solve $r - 6s + 9t = 0$.
11. Find PI of $(D^2 - 3DD' + 2D'^2)z = e^{x+y}$.
12. Find C.F of $(3D - 4D' - 2)z = 0$.

Section - B

Answer any five of the following :

(5×5=25)

13. Find the equation of the tangent plane to the surface $z = x^2 - y^2$ at the point $(2, -1, 3)$.
14. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r = xi + yj + zk$.

[P.T.O.]

15. Show that $\text{Curl}[r \times (a \times r)] = 3r \times a$ where a is a constant vector.
16. Using Green's theorem evaluate $\int_C e^{-x}(\sin y dx + \cos y dy)$ where C is the rectangle with vertices $(0,0), (\pi,0), (\pi, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$.
17. Using divergence theorem show that
- a) $\iiint_S r \cdot n ds = 3V$
- b) $\iiint_S \nabla r^2 \cdot ds = 6V$
18. Evaluate $\oint_C F \cdot dr$ where $F = 2yi + 3xj - z^2k$ using stoke's theorem, where C is the boundary of upper half of the surface of the sphere $x^2 + y^2 + z^2 = 9$.
19. Show that the external of $I = \int_{x_1}^{x_2} \sqrt{y[1+(y')^2]} dx$ is a parabola.

Section - C

Answer any five of the following.

(5×5=25)

20. Solve $(2D^2 - DD' - 3D'^2)z = 5e^{x-y}$.
21. Solve $(D^2 - 5DD' + 4D'^2)z = \sin(4x + y)$
22. Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$.
23. Solve $(D^2 - D'^2 + D - D')z = e^{x+3y}$.
24. Reduce the equation.

$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ into a canonical form.

25. Obtain the solution for one dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ by using the method of separation of variables.

