30522 (New)

B.Sc. V - Semester Degree Examination, Nov./Dec. - 2018

MATHEMATICS

Applied Mathematics

Paper - IX (5.2)

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

Answer All the sections.

Section - A

Answer any **Ten** of the following : **(10×2=20)**

1. If the vector $\overline{f} = (3x+3y+4z)i + (x-ay+3z)j + (3x+2y-z)k$ is solenoidal then find the constant 'a'.

2. Prove that $Curl\left(grad\phi\right) = 0$.

3. Find the directional derivative of the function $\overline{f} = x\sin z - y\cos z$ at the origin in the direction of 2i - 2j + k.

4. If $r = \left|\vec{r}\right|$ where $\vec{r} = xi + yj + zk$ show that $\nabla\vec{r} = \frac{1}{r}.\vec{r} = \hat{r}$.

5. Using Green's theorem, show that the area bounded by a simple closed curve 'c' is given by $\frac{1}{2}\oint_c [xdy - ydx]$.

6. Prove that $\frac{d}{dx}\left(f - y'\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial x} = 0$.

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7. Define Geodesics and write Euler - Poisson equation.

8. Write one - dimensional wave equation and write its appropriate solution.

9. Write the part of complementary function corresponding to

   i) non - repeated factor $(bD - aD' - c)$.

   ii) factor $(bD - aD')$.

10. Find CF of $(2D - 3D' - 4)^2 z = 0$.

11. Find PI of $\left[D^2 - 2DD' + (D')^2\right] z = e^{x+2y}$.

12. Solve $r - s + 6t = 0$.

Section - B

Answer any **Five** of the following : **(5×6=30)**

13. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1,2).

14. Prove that $Curl(Curl\overline{f}) = grad(div\overline{f}) - \nabla^2 \overline{f}$.

15. If $\vec{r} = xi + yj + zk$. Show that $r^n \vec{r}$ is irrotational for any constant 'n'.

16. Show that the following field $\overline{F}$ is a potential field and hence find its scalar potential ϕ. Such that $\overline{F} = \nabla\phi$. $\overline{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$.

. Verify Gauss divergence theorem for $F = 2xyi + yz^2j + xzk$ and S is the total surface of the rectangular parallelepiped bounded by the planes $x=0, y=0, z=0, x=1, y=2$ and $z=3$.

18. Verify stoke's theorem for the function $\overline{F} = y^2 i + xyj - xzk$ where 'S' is the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$.

19. Prove that the shortest distance between two points in a plane is a straight line.

Section - C

Answer any **Five** of the following : **(5×6=30)**

20. Find the curve on which the functional $\int_0^1 \left[(y')^2 + 12xy\right]dx$ with $y(0)=0$ and $y(1)=1$. Can be extremized.

21. Solve $\left[D^2 - 6DD' + 5(D')^2\right]z = 6x + 2y$.

22. Solve $\left[D^2 - DD' + D' - 1\right]z = \cos(x+2y) + e^y$.

23. Solve $\left[D^2 - 3DD' + 2(D')^2\right]z = e^{x+3y} + \sin(x-2y)$.

24. Obtain the solution for one - dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by using the method of separation of variables.

25. A string is stretched and fastend to two points distance 'l' apart, motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ and releasing from the rest. Show that the displacement of any point at a distance 'x' from one end at time 't' is given by $y(x,t) = a\sin\left(\frac{\pi x}{l}\right).\cos\left(\frac{\pi ct}{l}\right)$ for n = 1.