30423

B.Sc. IV Semester Degree Examination, June - 2018

MATHEMATICS - VIII

Topology and Real Analysis

Paper No. - 4.1

(OLD)

Time : 3 Hours   Maximum Marks : 60

Instructions to Candidates:

Answer **all** sections.

Section - A

Answer any **Ten** of the following :   (10×2=20)

1. Define indiscrete topology on a set.

2. Prove that a singleton set {x} is not an open set in (R,u).

3. Prove that the set Q of rationals is not closed in (R,u).

4. If A is closed and B is open then prove that A-B is closed.

5. Define interior and exterior of a set.

6. Prove that (R,u) is a T_1 - space.

7. Define compliment of a fuzzy subset.

8. Define inclusion of two fuzzy subsets.

9. Define partition of a closed interval.

10. Define Riemann integrable function and R - integral of a function.

11. State Darboux theorem.

12. Compute L [P,f] and U[P,f] if f(x) = x on [0,1] and $P = \left\{ 0, \dfrac{1}{3}, \dfrac{2}{3}, 1 \right\}$ be a partion of [0;1].

Section - B

Answer any **four** of the following :   (4×5=20)

13. Let (X, τ) be a topological space. $\mathcal{F}$ be family of all closed sets of X. then prove that

    1)   Any intersection of members of $\mathcal{F}$ is a member of $\mathcal{F}$.

    2)   Union of any two (hence a finite number of) members of $\mathcal{F}$ is a member of $\mathcal{F}$

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14. Define neighbourhood of a point. Let (X, τ) be a topological space. $x \in X$ if A is nhd of x then any super set of A is also nhd of x.

15. Let (X, τ) be a topological space Let A. B be subsets of X then prove that

    i) $d(\phi) = \phi$

    ii) $A \subset B \Rightarrow d(A) \subset d(B)$

16. Let (X, τ) be a discrete topological space. A be a subset of X find $\overset{o}{A}$, $(A')^o$, $\partial(A)$.

17. Let (X, τ) be a topological space, A,B be subsets of X then prove that

    1) $A \subset B \Rightarrow A^o \subset B^o$

    2) $(A \cap B)^o = A^o \cap B^o$

18. Define T_2 - space and prove that every T_2 space is a T_1 space.

Section - C

Answer any **four** of the following :   (4×5=20)

19. Let X be a set and A be a fuzzy subset of X let $\alpha, \beta \in [0,1]$ then prove that

    1) $\alpha \le \beta \Rightarrow \beta_A \subset \alpha_A$

    2) $\alpha \le \beta \Rightarrow \alpha_A \cup \beta_A = \alpha_A$

          $\alpha_A \cap \beta_A = \beta_A$

20. Let A,B be any two fuzzy subsets of X. Let $\alpha, \beta \in [0,1]$ Then prove that

    1) $\alpha_{(A \wedge B)} = \alpha_A \cap \alpha_B$

    2) $\alpha_{(A \vee B)} = \alpha_A \cup \alpha_B$

21. Prove that every monotonic function is R - Integrable in [a,b].

22. Show by an example that every bounded function need not be R - integrable.

23. If f is a bounded function and integrable in [a,b] prove that

    $$m(b-a) \le \int_a^b f(x)dx \le M(b-a) \; if \; a \le b$$

24. If f(x) is defined in [-1,1] as follows

    f(x) = 1 when x is a rational

         = -1 when x is a irrational

    then prove that f is not R - integrable in [a,b].