30424

IV Semester B.Sc. Degree Examination, September/October 2020

MATHEMATICS – VII

Paper 4.1 – Real Analysis and Complex Analysis

(CBCS – New)

Time : 3 Hours | Max. Marks : 60

Instructions : *Answer* ***all*** *Sections.*

SECTION – A

Answer **any ten** questions : **(10 × 2 = 20)**

1. Define upper and lower Darboux sums.
2. If $f, g \in R[a,b]$ then show that $f \cdot g \in R[a,b]$.
3. Prove that the lower Riemann integral cannot exceed upper Riemann integral.
4. If $f(x) = x$ on $[0, 1]$ and $P = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$. Find upper Darboux sum and lower Darboux sum.
5. Show that $\int_1^2 x^3\, dx = \frac{15}{4}$ by fundamental theorem of calculus.
6. Define Bilinear transformation.
7. Show that $\arg(\bar{z}/z) = \pi/2$ represents a line through the origin.
8. Show that $f(z) = e^z$ satisfies ER equations.
9. Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.
10. Evaluate $\int_C (\bar{z})^2\, dz$ around the circle $|z| = 1$.
11. Evaluate $\int_0^{3+i} z^2\, dz$ along the line $3y = x$.
12. Evaluate $\int_C \frac{1}{z(z-1)}\, dz$ where 'C' is $|z| = 2$.

SECTION – B

Answer **any two** of the following : **(2 × 5 = 10)**

13. If $f(x)$ is continuous function defined on $[a, b]$ then prove that $f(x)$ is R-integrable.

14. State and prove fundamental theorem of integral calculus.

15. If $f \in R\,[a, b]$ then prove that $|\,f\,| \in R\,[a, b]$.

SECTION – C

Answer **any three** questions : **(3 × 5 = 15)**

16. State and prove sufficient condition for $f(z)$ to be analytic.

17. Find the analytic function $f(z) = u + iv$ given that $u + v = e^x(\cos y + \sin y)$.

18. Prove that $f(z) = \cosh z$ is analytic and hence show that $f(z) = \sinh z$.

19. If the real part of an analytic function is $(r^2 \cos 2\theta - r \sin\theta)$ then find the corresponding imaginary part.

SECTION – D

Answer **any three** questions : **(3 × 5 = 15)**

20. State and prove Cauchy's integral formula.

21. Evaluate $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)}\,dz$ where 'C' is the circle $|\,z\,| = 1.5$.

22. Evaluate $\int_0^{1+i} (x^2 - iy)\,dz$ along $y = x$ and $y = x^2$.

23. Find bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$. Also find the fixed points of the transformation.