

B.Sc. IV Semester Degree Examination, May/June-2019

MATHEMATICS

Real Analysis and Complex Analysis

Paper No. - VII 4.1

(New)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all Sections.

SECTION - A

Answer any Ten questions.

(10×2=20)

1. Define upper and lower Darboux sums.
2. Compute $L(p, f)$ and $U(p, f)$, if $f(x) = x^2$ for $x \in [0, 3]$ and let $P = \{0, 1, 2, 3\}$ be the partition of $[0, 3]$.
3. State Darboux theorem.
4. If P_1 and $P_2 \in f[a, b]$, then prove that $L(P_1, f) \leq U(P_2, f)$.
5. Show that $\int_a^b e^x dx = e^b - e^a$ by using fundamental theorem of calculus.
6. Define conformal transformation.
7. Verify whether $f(Z) = Z - \bar{Z}$ is differentiable or not by using Cauchy-Rieman equation.
8. Show that, $\arg(Z - 1) = \frac{\pi}{2}$ represents a line parallel to imaginary axis.
9. Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.

[P.T.O]

10. Show that $f(z) = u + iv$ is analytic. If $u = \frac{1}{2} \log(x^2 + y^2)$ and $V = \tan^{-1}\left(\frac{y}{x}\right)$.
11. Evaluate $\int_C \frac{\cos \pi z}{(z-1)} dz$, where C is $|z| = \frac{3}{2}$.
12. Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along $y = x^2$.

SECTION - B

Answer any Two of the following.

(2×5=10)

13. Prove that a constant function defined on a closed interval is Riemann integrable.
14. If $f \in R[a, b]$ and m, M are respectively the infimum and supremum of $f(x)$ on (a, b) then prove that $m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$.
15. If $f(x)$ is Riemann integrable and $\phi(x)$ is primitive of $f(x)$, then prove that $\int_a^b f(x) dx = \phi(b) - \phi(a)$.

SECTION - C

Answer any Three questions..

(3×5=15)

16. State and prove the C-R equations in Polar form.
17. Show that $f(z) = \sin hz$ is an analytic function and find its derivative using C-R equations.
18. Find the analytic function $f(z) = u + iv$ given that $V(x, y) = x \sin x \sin hy - y \cos x \cos hy$.
19. Show that $v = r \sin \theta + \frac{\cos \theta}{r}$ ($r \neq 0$) is harmonic. Find the analytic function, whose imaginary part is $v(r, \theta)$ by Milne-Thomson's method.



SECTION - D

Answer any **Three** questions.

(3×5=15)

20. State and prove Cauchy's integral theorem.

21. Evaluate $\int_c \frac{z}{(z^2 + 1)(z^2 - 9)} dz$, where 'c' is the circle $|z| = 2$.

22. Evaluate $\int_c (\bar{z})^2 dz$ around the circle $|z - 1| = 1$.

23. Find the Bilinear transformation which maps the points $z=0, -i, 2i$ into $w = 5i, \infty, -i/3$.
Also find invariant points of the transformation.
