30276

B.Sc. II - Semester Degree Examination, June - 2018

MATHEMATICS

Mathematics - III (Algebra - II)

Paper No : 2.1

(New)

Time : 3 Hours Maximum Marks : 60

Instructions to Candidates:

*Answer **all** sections.*

Section - A

Answer any **Ten** of the following : **(10×2=20)**

1. If every element of group G its own inverse then show that G is abelian.
2. Show that multiplicative group of cube roots of unity is a cyclic group.
3. Find the number of generators of the cyclic group G of order 60.
4. Consider multiplicative group G = {1, -1, i, -i} of fourth roots of unity & H = {-1, 1} be a subgroup of G. Write all right cosets of H in G.
5. Find lower bound and upper bound of the sequence $\left\{2^{(-1)^n}\right\}$.
6. Discuss the nature of the sequence whose n^{th} term is $\sqrt{n+1}-\sqrt{n}$.
7. Show that the sequence $\left\{\frac{1}{n}\right\}$ is monotonically decreasing.
8. If a series $\sum u_n$ is convergent then show that $\lim_{n\to\infty} u_n = 0$.
9. Define convergent and divergent sequences with example for each.
10. Discuss the convergence of the series $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots$

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11. Discuss the convergence of the series $\sum \sin \frac{1}{n}$.

12. State D'Alemberts ratio test.

Section - B

Answer any **Two** of the following : **(2×5=10)**

13. State and prove Lagranges theorem.

14. Prove that the set G/H of all cosets of H where H is normal subgroup of G under the binary operation defined by Ha.Hb=Hab $\forall Ha, Hb \in G/H$ is a group.

15. Define homomarphism of groups. If $f : G \to G'$ is a homomarphism from the group G into G' with kernel k, then show that f is one - one if and only if k = {e} where e is the identity element of G.

Section - C

Answer any **Three** of the following : **(3×5=15)**

16. Show that limit of convergent sequence is unique.

17. If $\lim_{n\to\infty}\{x_n\} = l$, $\lim_{n\to\infty}\{y_n\} = m$ then show that $\lim_{n\to\infty}\{x_n - y_n\} = l - m$.

18. Prove that a monotonic decreasing sequence bounded below is convergent.

19. Show that the sequence $\{x_n\}$ where $x_n = \frac{3n+4}{2n+1}$ is

    i) monotonic decreasing

    ii) bounded

    iii) tends to the limit $\frac{3}{2}$

Section - D

Answer any **Three** questions. **(3×5=15)**

20. Show that the nature of the series is not altered by the multiplication of all the terms of the series by the same non - zero constant c.

21. Discuss the convergence of series

$$1+\frac{2!}{2^n}+\frac{3!}{3^n}+\frac{4!}{4^n}+.....$$

22. Show that the Geometric series $\sum x^n$

i) Converges if $|x|<1$ ie if $-1 < x < 1$.

ii) Diverges if $x \geq 1$

iii) Oscillates if $x \leq -1$

23. Discuss the convergence of

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+.....$$