



**II Semester B.Sc. Degree Examination, May/June 2016**  
**MATHEMATICS – II**  
**Paper – 2.2 : Advanced Calculus (New)**

Time : 3 Hours

Max Marks 60

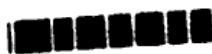
**Instruction** Answer all Sections

**SECTION – A**

**Answer any ten of the following :**

**(10x2=20)**

1. State Rolle's theorem for differential calculus.
2. Verify Cauchy's mean value theorem for the functions  $f(x) = \sqrt{x}$  and  $g(x) = \frac{1}{\sqrt{x}}$  in  $[a, b]$ .
3. Verify Lagrange's mean value theorem for  $f(x) = (x - 1)(x - 2)(x - 3)$  in  $[0, 4]$ .
4. Evaluate  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^2}$ .
5. Evaluate  $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$ .
6. Evaluate  $\lim_{x \rightarrow 0} (1 - x^2)^{\frac{1}{\ln(1-x)}}$ .
7. Evaluate  $\int \int [3x + y] dx + (2y - x) dy$  along  $y = x^2 + 1$  from  $(0, 1)$  and  $(3, 10)$ .
8. Evaluate  $\int \int \int (x^2 + y^2) dy dx$ .
9. Evaluate  $\int \int \int xyz dx dy dz$ .
10. Evaluate  $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \cos^2(\alpha) d\alpha$ .
11. Show that  $\Gamma(n+1) = n \Gamma(n)$ .
12. Evaluate  $\int_0^{\infty} \cos^2(\alpha) d\alpha$  using Beta and Gamma functions.



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### SECTION - B

(5x5=25)

Answer any five of the following :

13. State and prove Cauchy's mean value theorem.
14. Expand  $\log [x + \sqrt{1+x^2}]$  upto first four terms by Maclaurin's expansion.
15. If  $x > 0$ , show that  $0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1$ .
16. Obtain the reduction formula for  $\int_0^{\pi/2} \sin^n x dx$
17. Find the area of the loop of the curve  $ay^2 = x^2(a-x)$ . The curve passes through  $(0, 0)$ .
18. Find the volume of the solid formed by revolving one arch of the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ , about the  $x$ -axis
19. Find the perimeter of the cardioid  $a(1 + \cos \theta) = r$  is symmetric about the initial line.

### SECTION - C

Answer any three of the following :

(3x5=15)

20. Evaluate  $\int_C [(x+2y)dx + (4-2x)dy]$  around the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  in the counter clockwise.
21. Evaluate  $\iint_R xy dx dy$ , where  $R$  is the quadrant of the circle  $x^2 + y^2 = a^2$ , where  $x \geq 0, y \geq 0$ .
22. Evaluate  $\iiint_{000}^{111} e^{x+y+z} dx dy dz$ .
23. Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$  and  $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$
24. Show that  $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m, n)$ .