



15114

**First Semester B.Sc. Degree Examination, Nov./Dec. 2016**

**MATHEMATICS – II**

**Paper – 1.2 : Calculus – I (Old)**

Time : 3 Hours

Max. Marks : 60

**Instruction : Answer all Sections.**

**SECTION – A**

Answer **any ten** of the following :

**(10×2=20)**

1. If  $f(x) = \frac{x^2 - 9}{x - 3}$  find  $\lim_{x \rightarrow 3} f(x)$ .
2. Discuss the continuity of the function  $f(x) = \sin \frac{1}{x}$  at  $x = 0$ .
3. Find the  $n^{\text{th}}$  derivative of  $\cos(ax + b)$ .
4. Find the angle between the radius vector and the tangent for the curve  
 $r = a(1 + \sin \theta)$  at  $\theta = \frac{\pi}{4}$ .
5. Show that the pairs of curves intersect orthogonally  $r = a\theta$ ,  $r = \frac{a}{\theta}$ .
6. For the cardioid  $r = a(1 - \cos \theta)$  S.T  $2ap^2 = r^3$ .
7. Define radius of curvature and write the formula for radius of curvature in Cartesian and polar forms.

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8. Find the co-ordinates of the centre of curvature at  $(x, y)$  for the curve

$$y = a \cosh \left( \frac{x}{a} \right).$$

9. Find the envelope of the family of circles  $(x - \alpha)^2 + y^2 = \alpha^2$  where  $\alpha$  is a parameter.

10. Find the asymptotes parallel to co-ordinate axes  $x^3 - y^2x = y^2 + 1$ .

11. Show that  $y = e^x$  is everywhere concave upwards.

12. Define multiple points and double point of the curve.

### SECTION – B

Answer **any four** of the following :

**(4×5=20)**

13. Find the  $n^{\text{th}}$  derivative of

a)  $\cos x \cdot \cos 2x \cdot \cos 3x$

b)  $\text{Log}(ax + x^2)$ .

14. If  $\cos^{-1} \left( \frac{y}{b} \right) = \log \left( \frac{x}{n} \right)^n$  show that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$ .

15. Find the  $n^{\text{th}}$  derivative of  $e^{ax} \cos(bx + c)$ .

16. Show that the curves  $r^2 = a^2 \cos 2\theta$  and  $r = a(1 + \cos \theta)$  intersect at an angle

$$3 \sin^{-1} \left( \frac{3}{4} \right)^{1/4}.$$

17. Show that the pedal equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \frac{a^2 b^2}{p^2} + r^2 = a^2 + b^2.$$

18. Derive formula for arc length in polar form.



SECTION – C

Answer **any four** of the following :

(4×5=20)

19. Show that the radius of curvature of the curve  $x^3 + y^3 = 3axy$  at point

$$\left(\frac{3a}{2}, \frac{3a}{2}\right) \text{ is } \frac{3a}{8\sqrt{2}}.$$

20. Find the evolute of the parabola  $y^2 = 4ax$ .

21. Find the envelope of the family of lines  $\frac{x}{a} + \frac{y}{b} = 1$  where 'a' and 'b' are connected by the relation  $ab = c^2$ .

22. Find the range of x for which the following curves are concave upwards or downwards ?

$$y = 3x^5 - 40x^3 + 3x - 20.$$

23. Find all the asymptotes of the curve :

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0.$$

24. Trace the curve strophoid  $y^2 = x^2 \left( \frac{a+x}{a-x} \right)$ .

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